Advanced Programming

Fall 97

University of Tehran

Recursion

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What is Recursion?

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- Usually recursion involves a function **calling itself**.

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- A recursive algorithm must **change its state** and move **toward the base case**. (Avoiding loops)
- A recursive algorithm must **call itself**, recursively.

1. Integer Factorial

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unsigned int factorial(unsigned int n) {
    if(n == 0)
        return 1;
    else
        return n * factorial(n - 1);
}
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Recursive Call

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Do Sth with Result of Subproblem

Recursive Call

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```
unsigned int fibonacci(unsigned int n) {
    if(n == 1 || n == 2)
        return 1;
    else
        return fibonacci(n - 1) + fibonacci(n - 2);
}
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Multiple Base Cases

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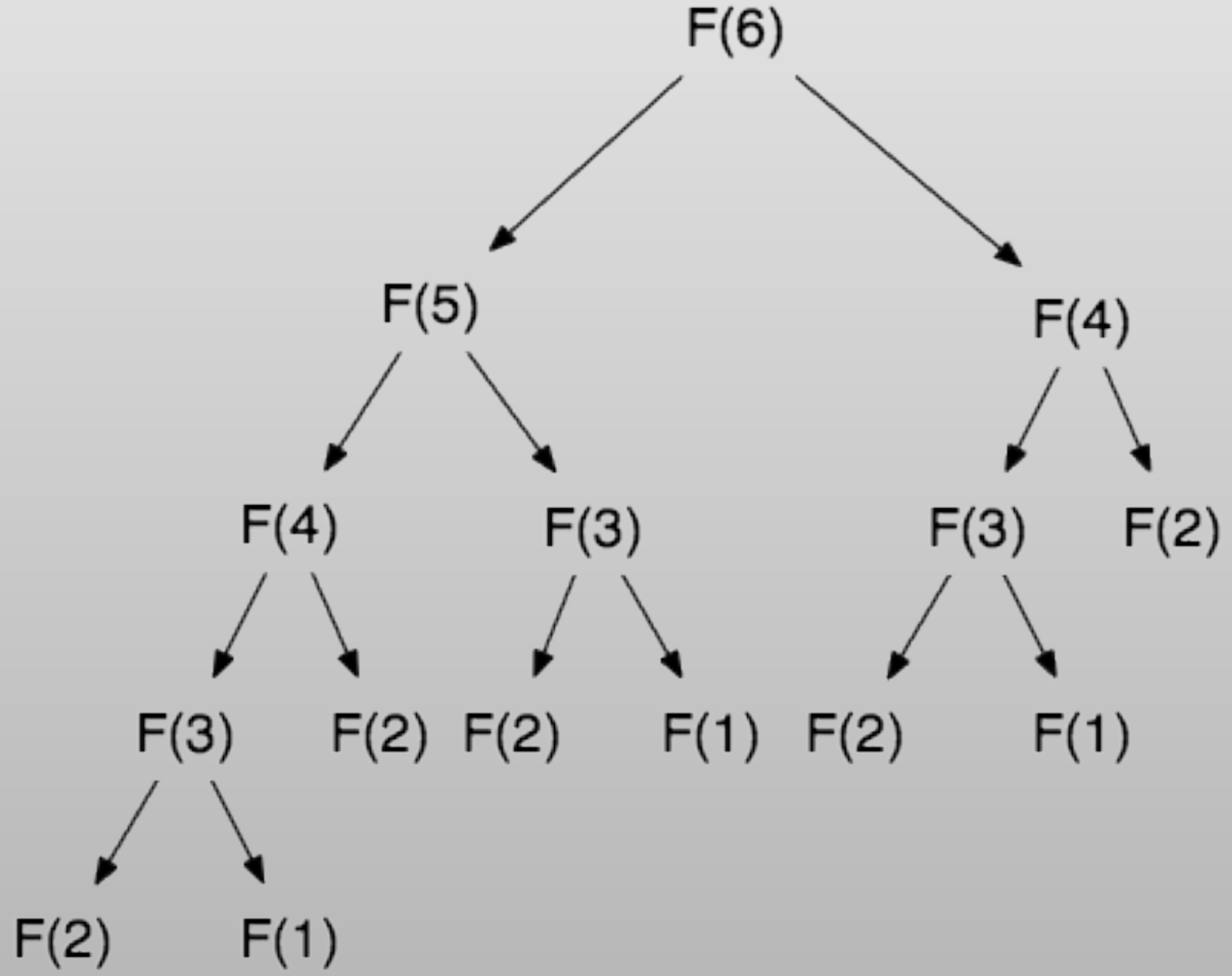
Do Sth with Result of Subproblems

Multiple Recursive Call

2. Fibonacci Seq.



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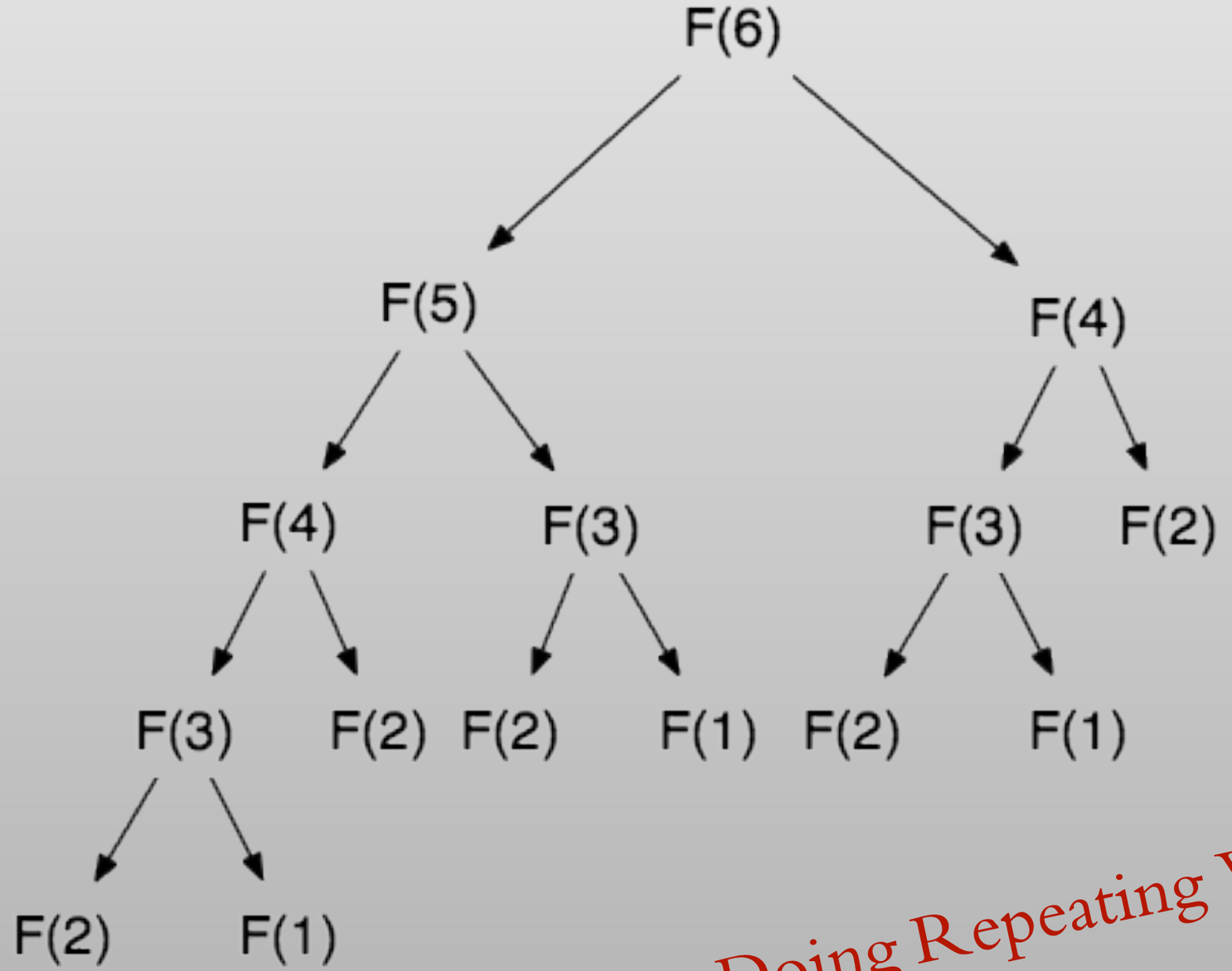

Doing Repeating Work ?

3. Converting an Integer to a String in Any Base

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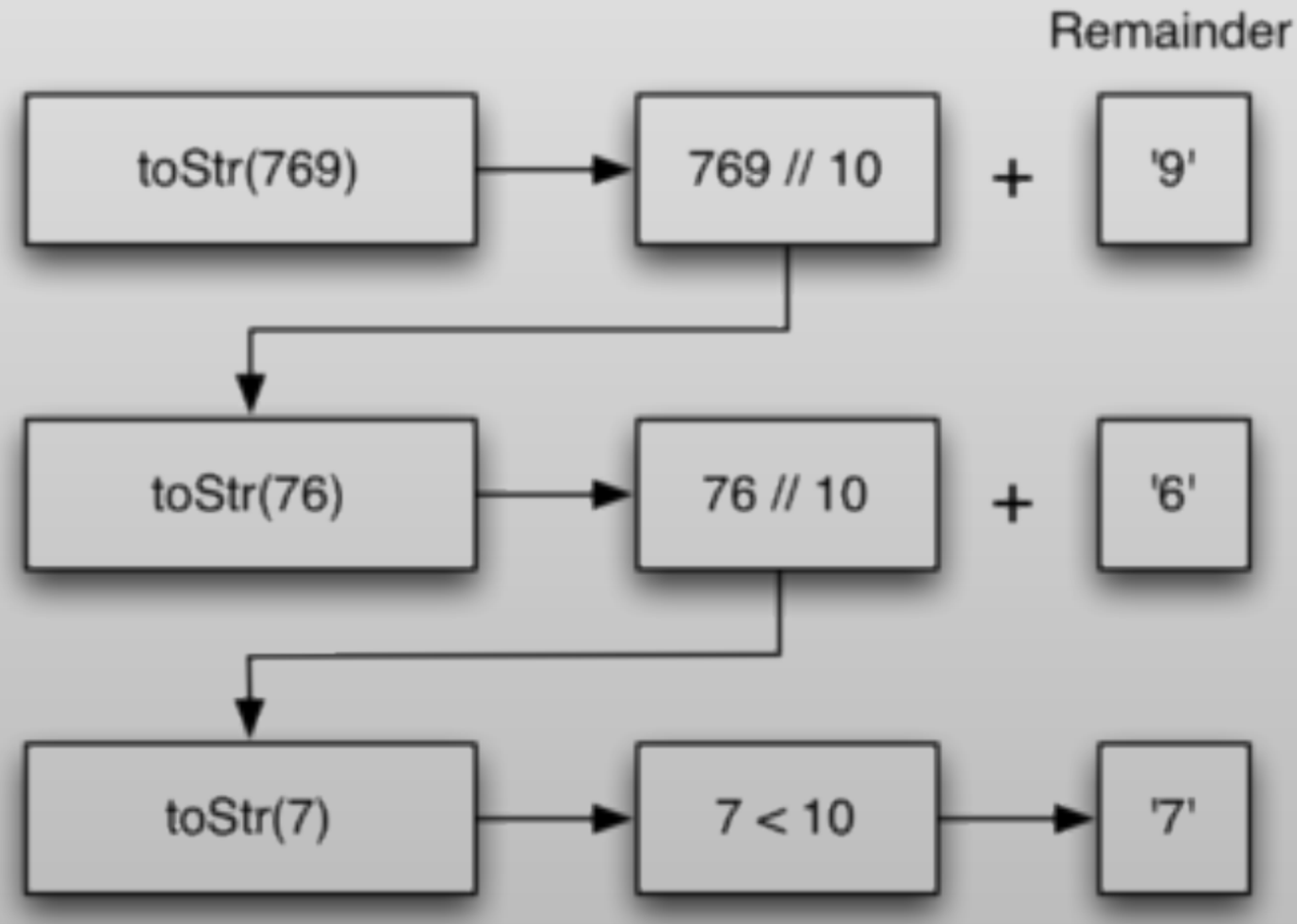

3. Converting an Integer to a String in Any Base

```
string to_string(unsigned int n, unsigned int base) {
    static const string digits = "0123456789ABCDEF";
    if(base > digits.size())
        throw invalid_argument("invalid base");
    if(n < base)
        return string(digits[n], 1);
    else
        return to_string(n / base, base) + string(digits[n % base], 1);
}
```

4. Binary Search

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int binary_search(vector<int> list, int key);
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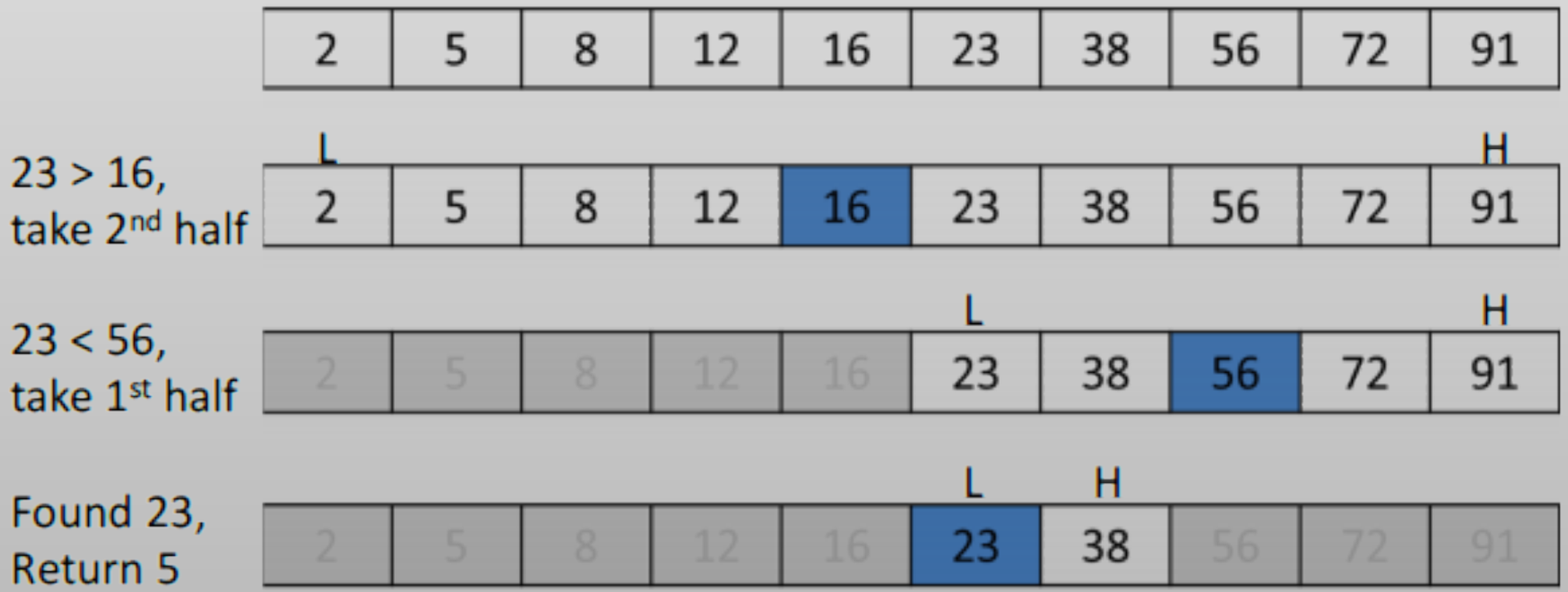
If searching for 23 in the 10-element array:



4. Binary Search

```
int binary_search(vector<int> list, int from, int to, int key)
{
    if (list.size() == 0 || from > to)
        return -1;

    int mid = (from + to) / 2;
    if (list[mid] == key)
        return mid;
    else if (list[mid] < key)
        return binary_search(list, mid + 1, to, key);
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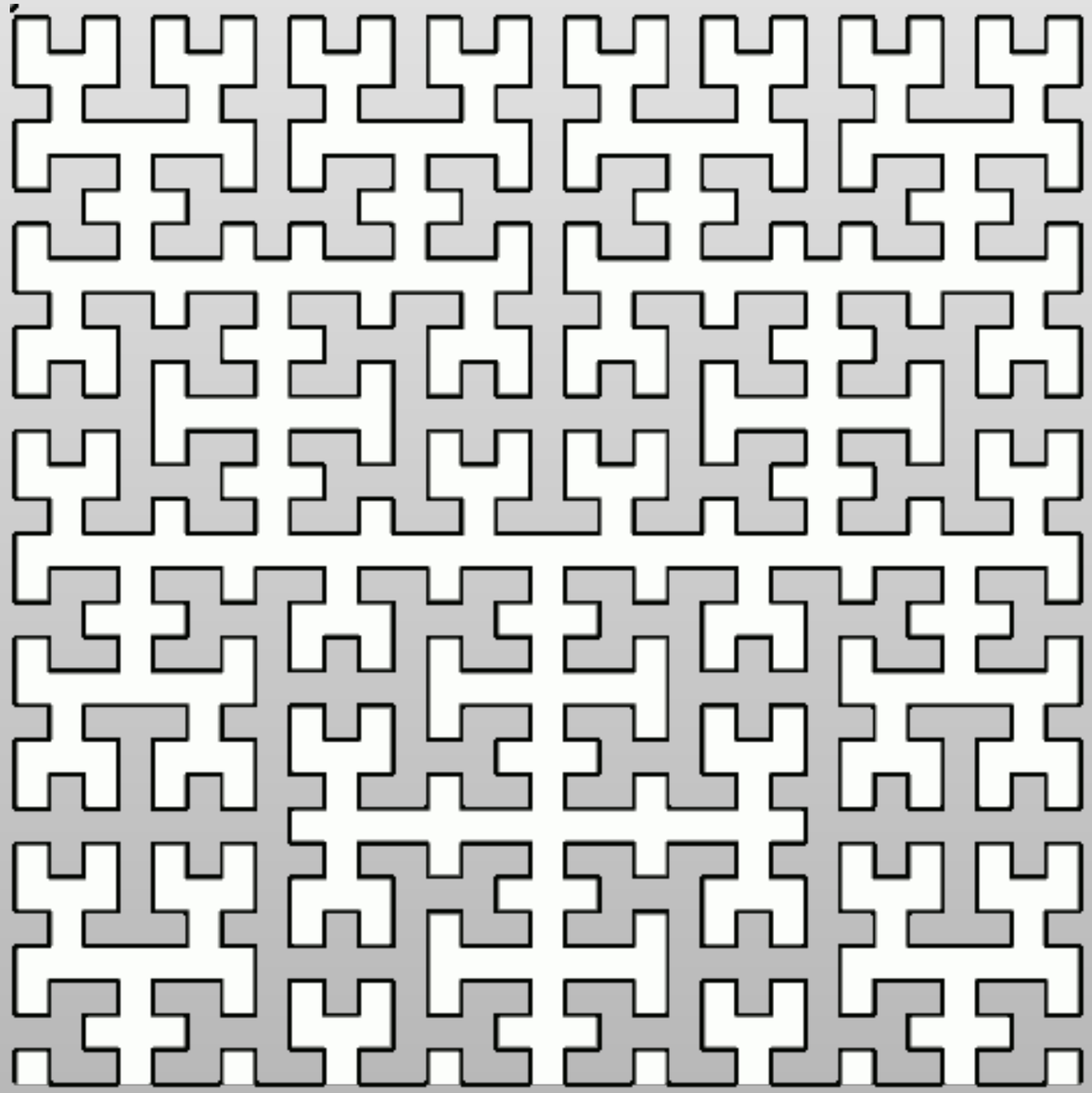
    else
        return binary_search(list, from, mid - 1, key);
}
int binary_search(vector<int> list, int key) {
    return binary_search(list, 0, list.size() - 1, key);
}
```

Solving More General Problem

5. Hilbert Curve



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David Hilbert

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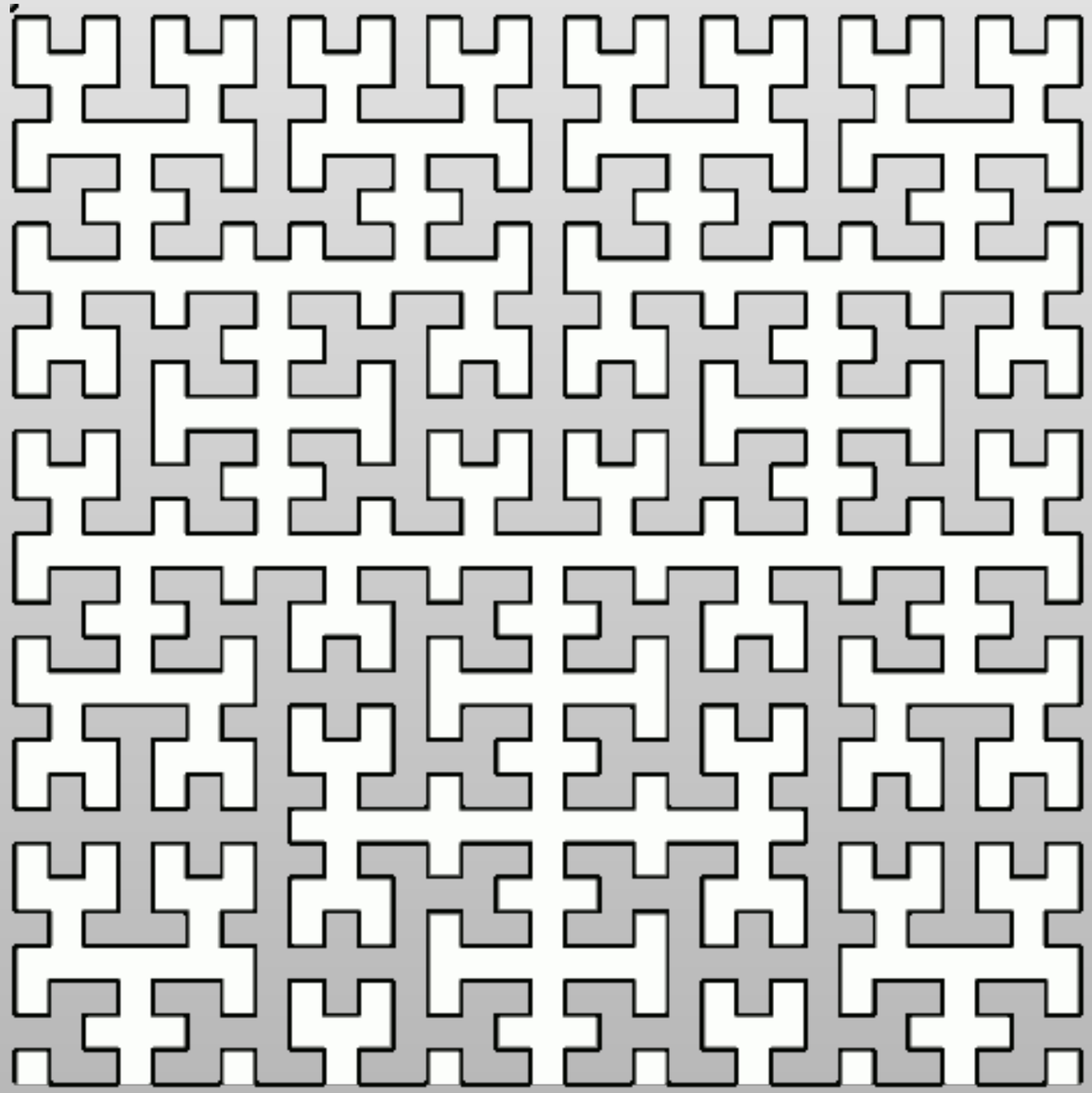

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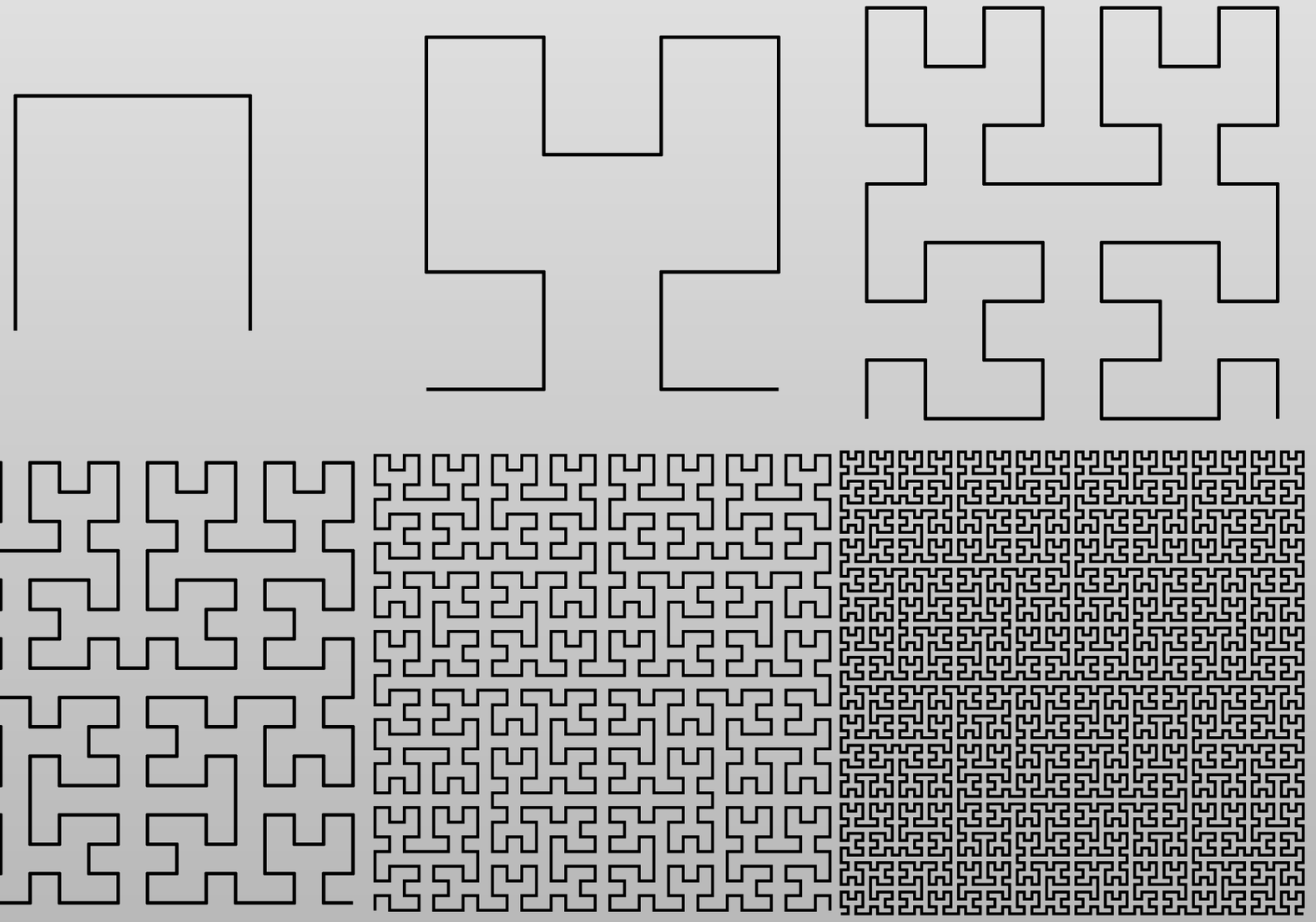

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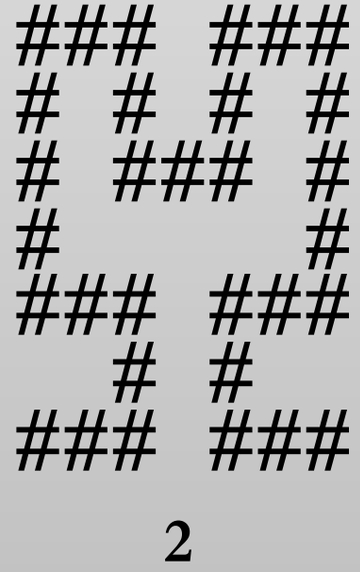

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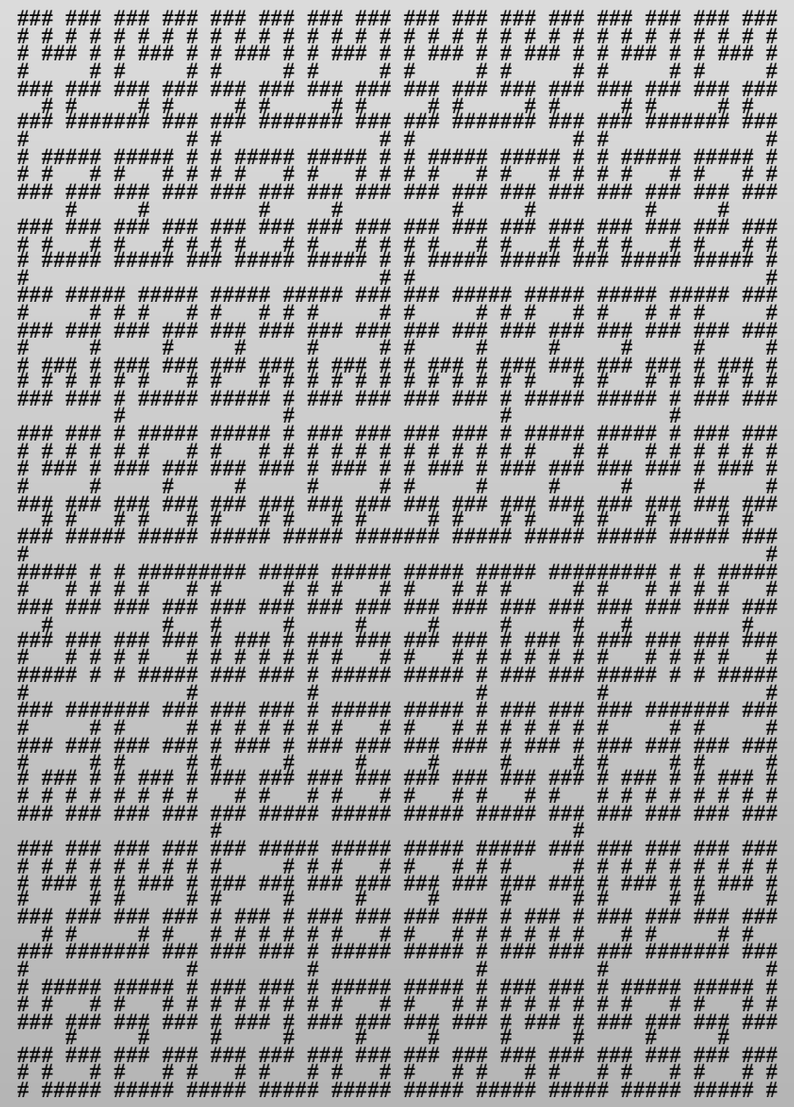

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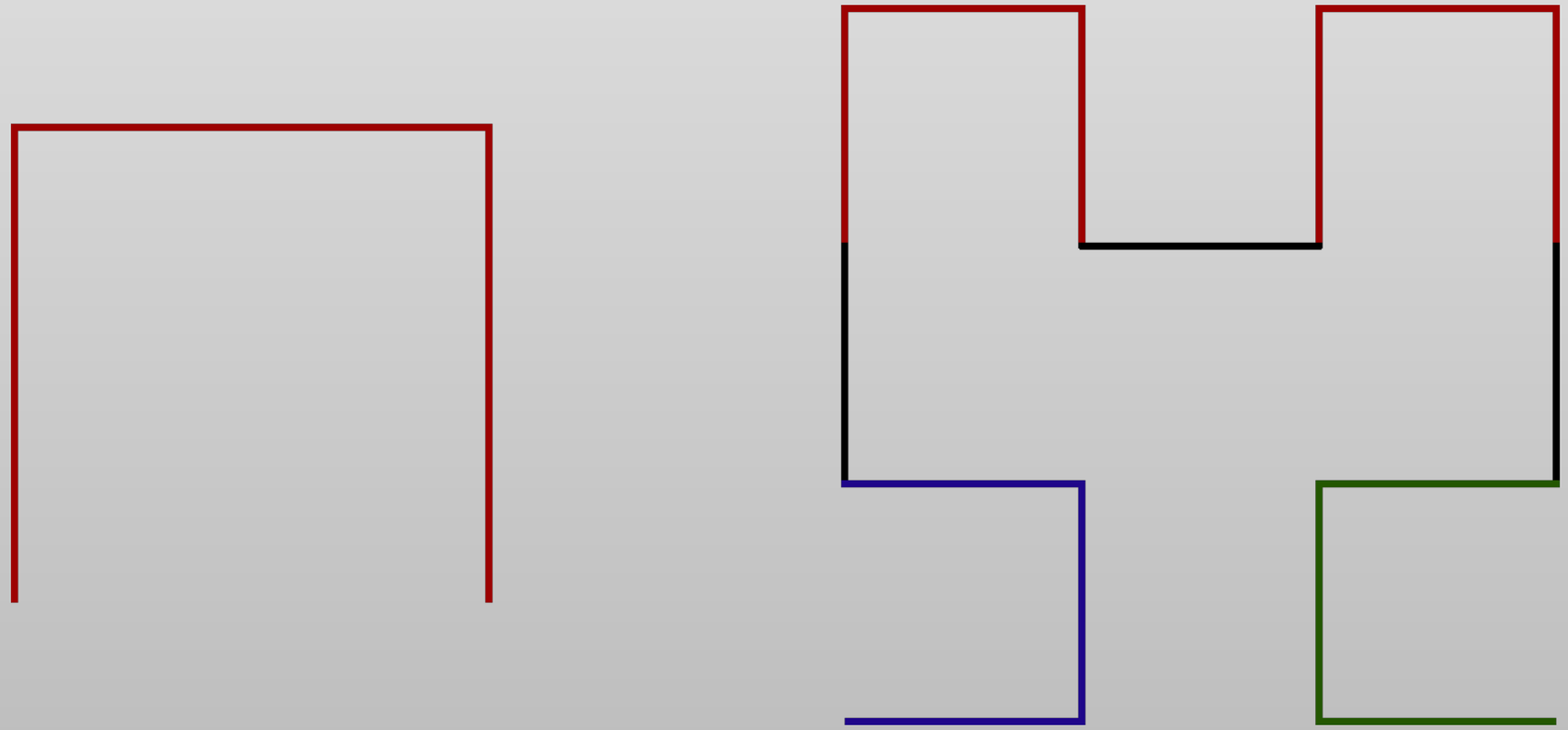

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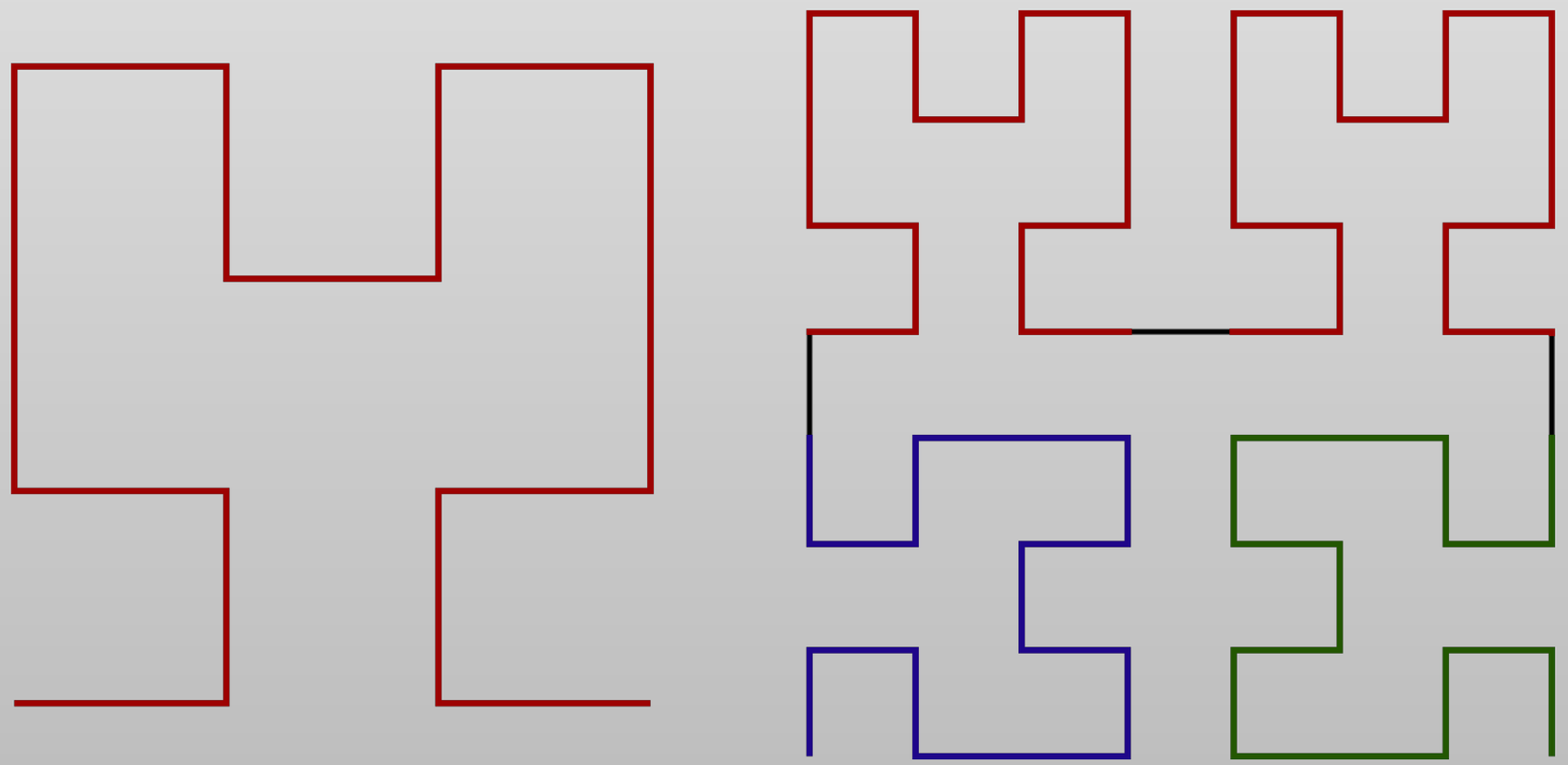

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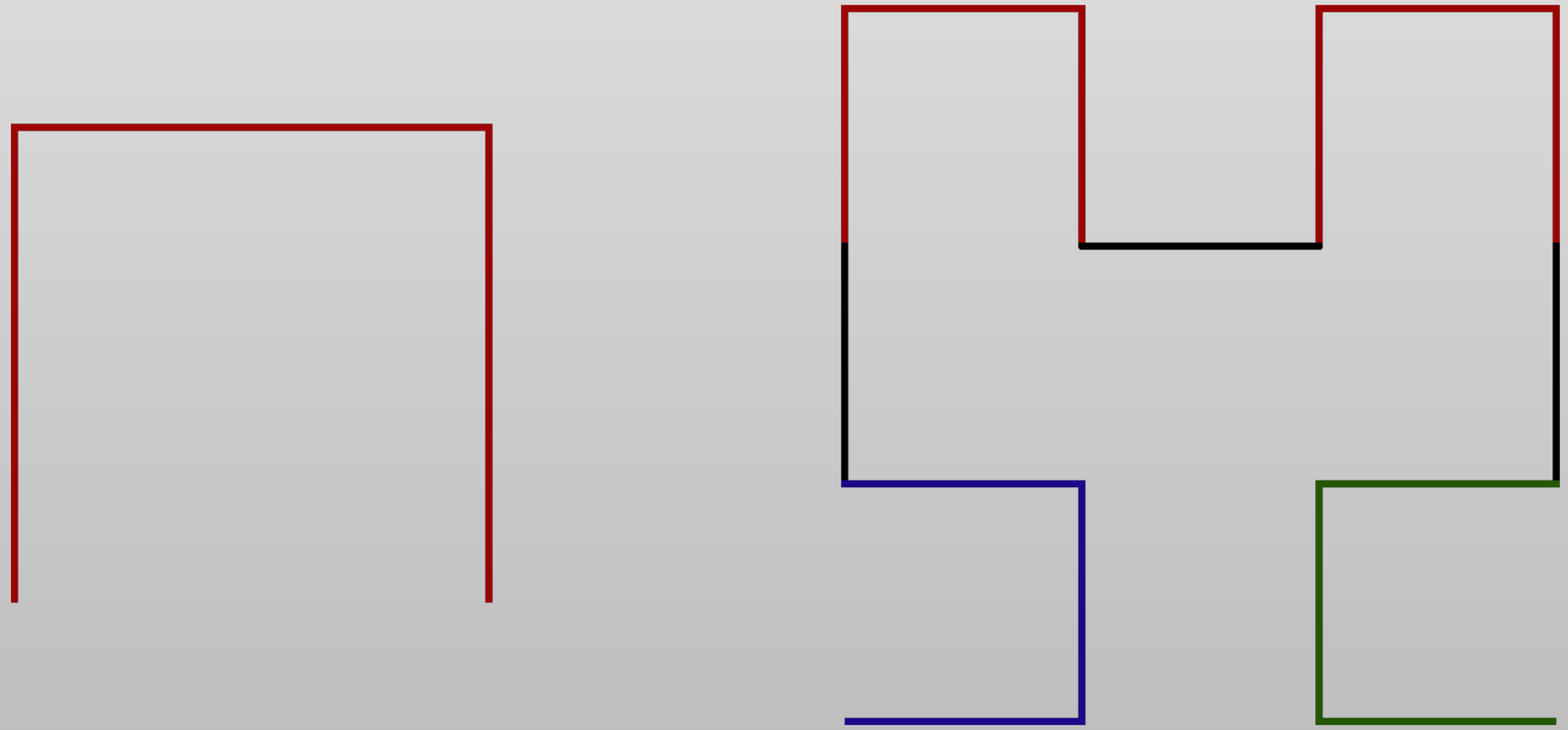

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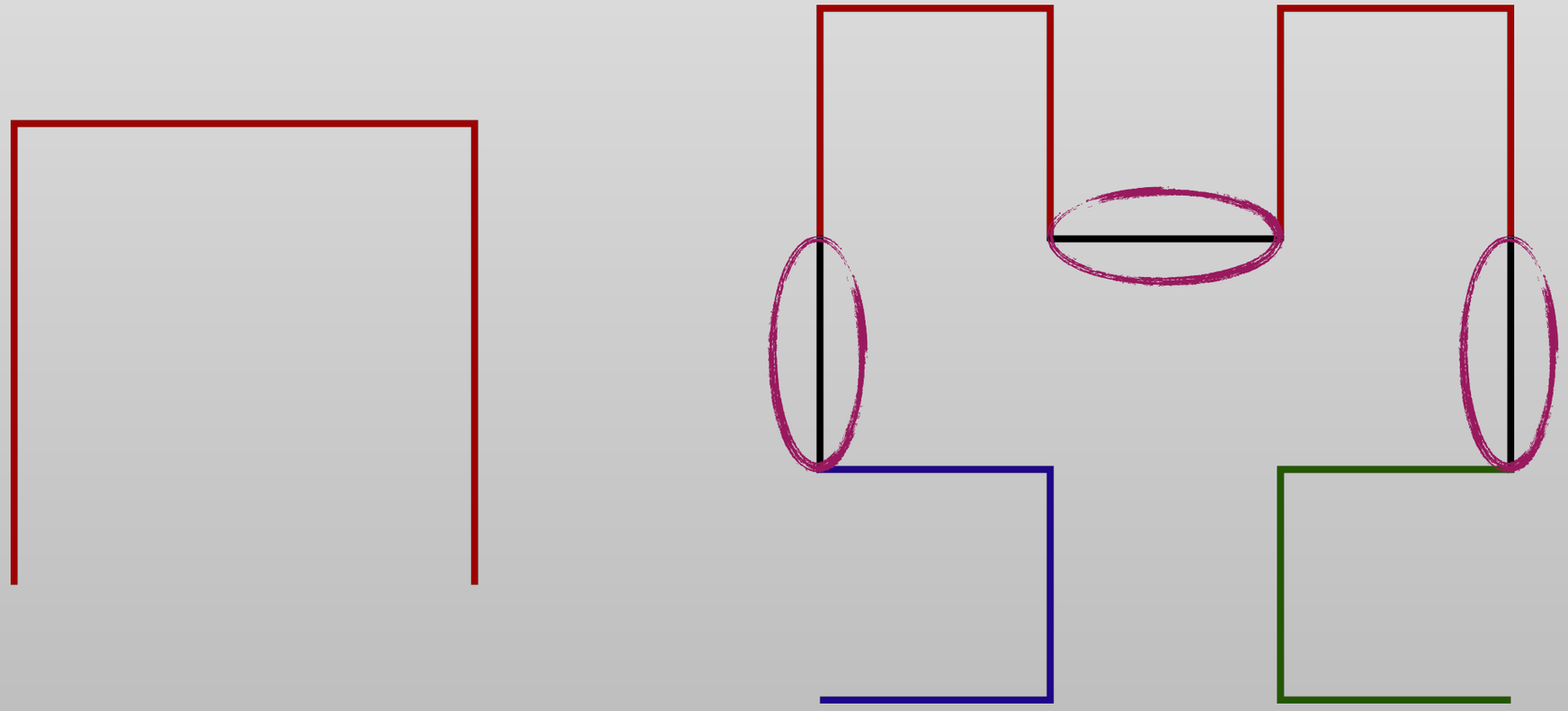

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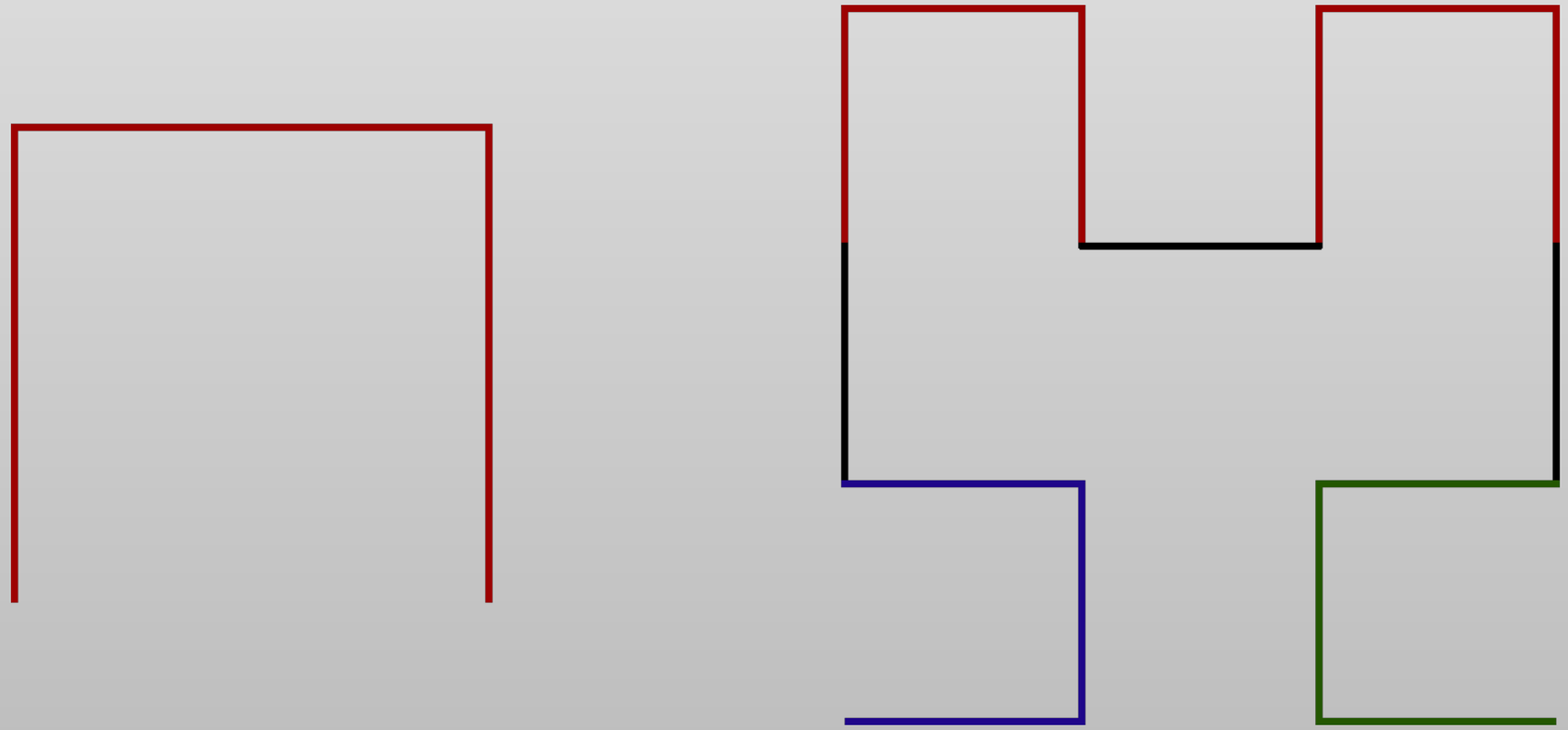

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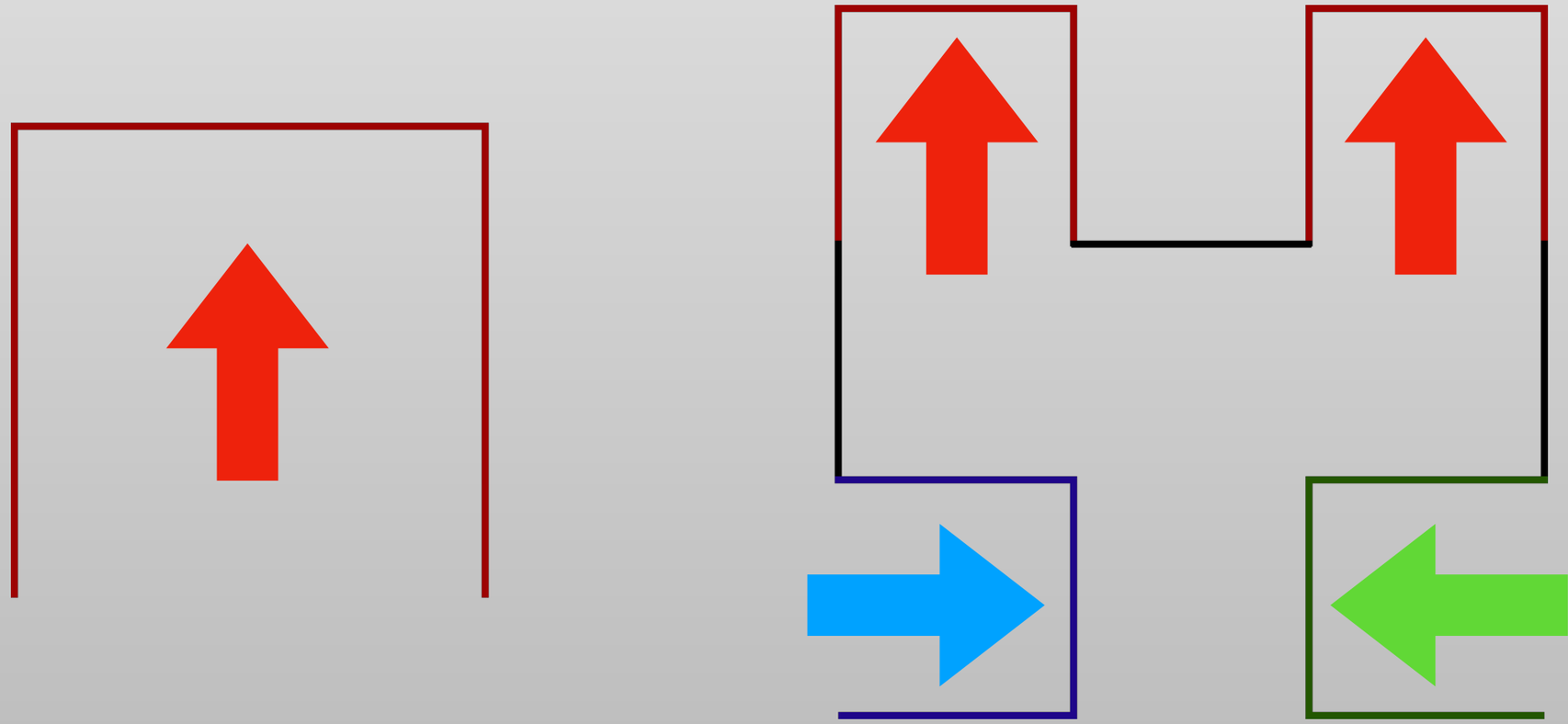

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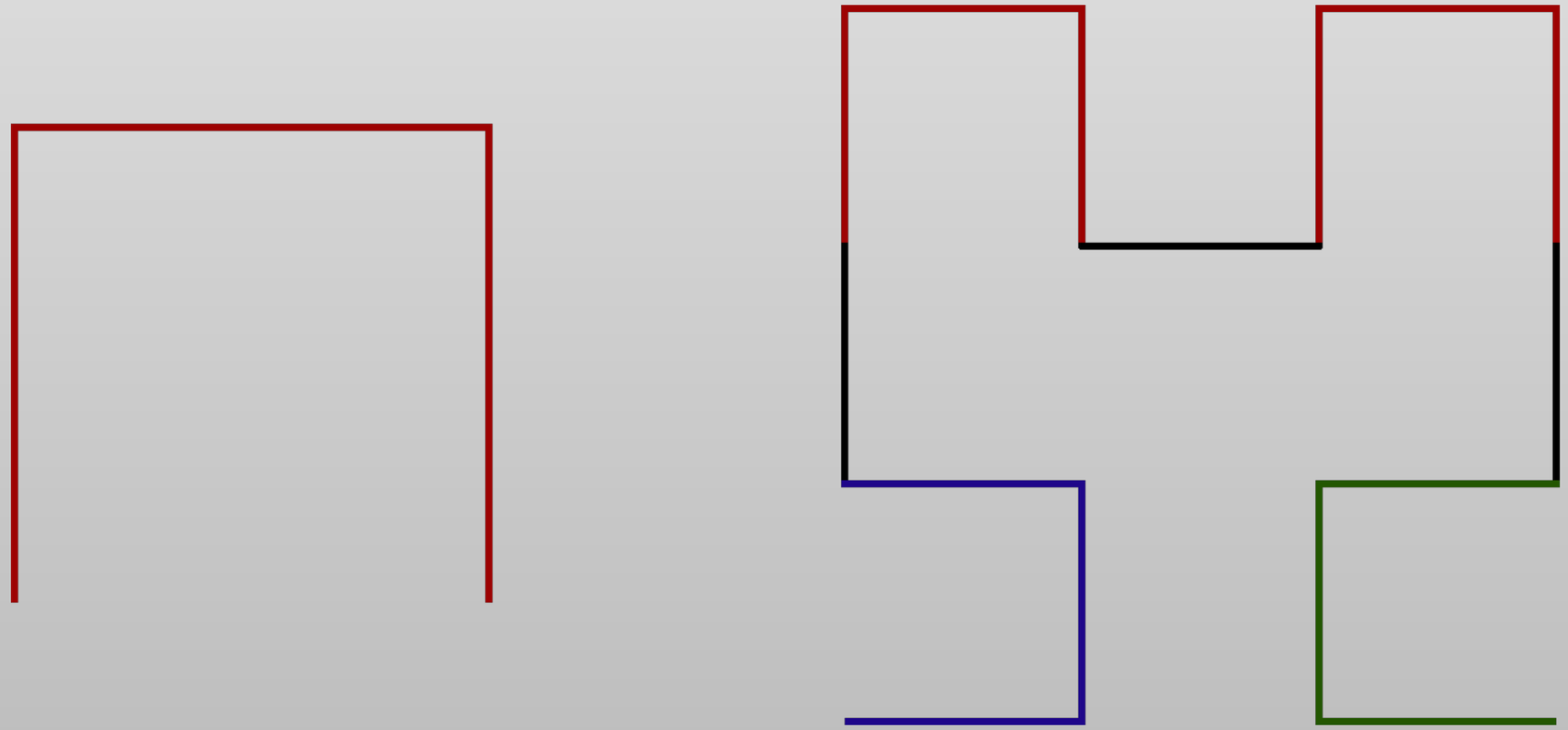

5. Hilbert Curve

```
typedef pair<int, int> Coord;
typedef vector<bool> Row;
typedef vector<Row> Matrix;
enum Direction { Up, Down, Right, Left };
```

5. Hilbert Curve

```
const Direction recursive_directions[4][2][2] = {
    {{Up, Up}, {Right, Left}},      // Up
    {{Left, Right}, {Down, Down}}, // Down
    {{Down, Right}, {Up, Right}},  // Right
    {{Left, Down}, {Left, Up}}     // Left
};
```

5. Hilbert Curve

```
int get_grid_length(int n) {
    return pow(2, n) - 1;
}
```

5. Hilbert Curve

```
void make_grid(unsigned int n, Matrix &grid, Coord start, Direction direction) {
  if (n == 1) {
    grid[start.first][start.second] = true;
    return;
  }
  int length = get_grid_length(n);
  if (direction == Up || direction == Down) {
    grid[start.first + length / 2][start.second] = true;
    grid[start.first + length / 2][start.second + length - 1] = true;
    grid[start.first + length / 2 + (direction == Up ? -1 : 1)]
        [start.second + length / 2] = true;
  } else {
    grid[start.first][start.second + length / 2] = true;
    grid[start.first + length - 1][start.second + length / 2] = true;
    grid[start.first + length / 2]
        [start.second + length / 2 + (direction == Left ? -1 : 1)] = true;
  }
  make_grid(n - 1, grid, start, recursive_directions[direction][0][0]);
  make_grid(n - 1, grid, Coord(start.first, start.second + length / 2 + 1),
            recursive_directions[direction][0][1]);
  make_grid(n - 1, grid, Coord(start.first + length / 2 + 1, start.second),
            recursive_directions[direction][1][0]);
  make_grid(n - 1, grid,
            Coord(start.first + length / 2 + 1, start.second + length / 2 + 1),
            recursive_directions[direction][1][1]);
}
```

5. Hilbert Curve

```
Matrix make_grid(unsigned int n) {
  Matrix grid(get_grid_length(n), Row(get_grid_length(n), false));
  make_grid(n, grid, Coord(0, 0), Up);
  return grid;
}
```

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```

References

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